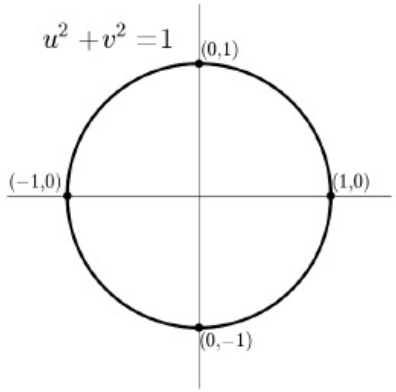
Congruent Number Problem

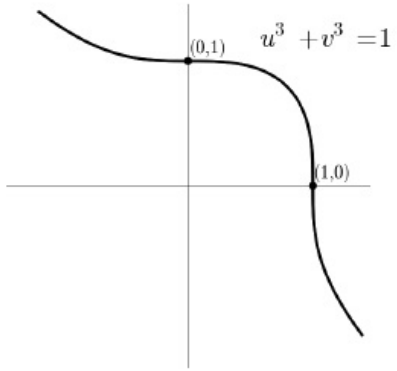
Hikmet Burak Özcan

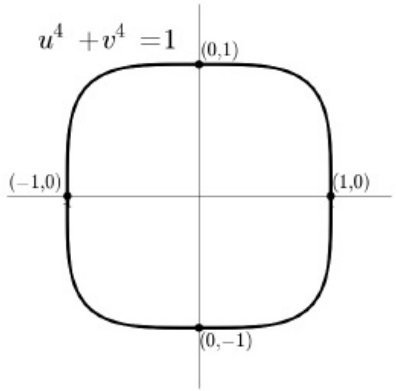
Adnan Menderes University

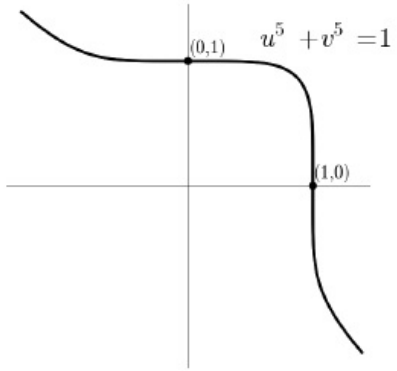
hikmetburakozcan@gmail.com

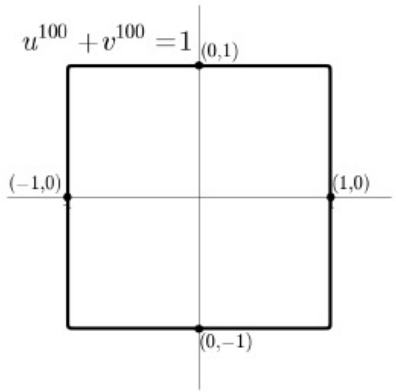
23/11/2018

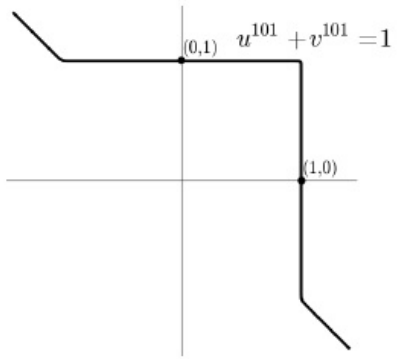

$u^2+v^2=1$
(0,1)
(−1,0)
(1,0)
(0,−1)


$u^3+v^3=1$
(0,1)
(1,0)


$u^4+v^4=1$
(0,1)
(−1,0)
(1,0)
(0,−1)


$u^5+v^5=1$
(0,1)
(1,0)


$u^{100}+v^{100}=1$
(0,1)
(−1,0)
(1,0)
(0,−1)


$u^{101}+v^{101}=1$
(0,1)
(1,0)

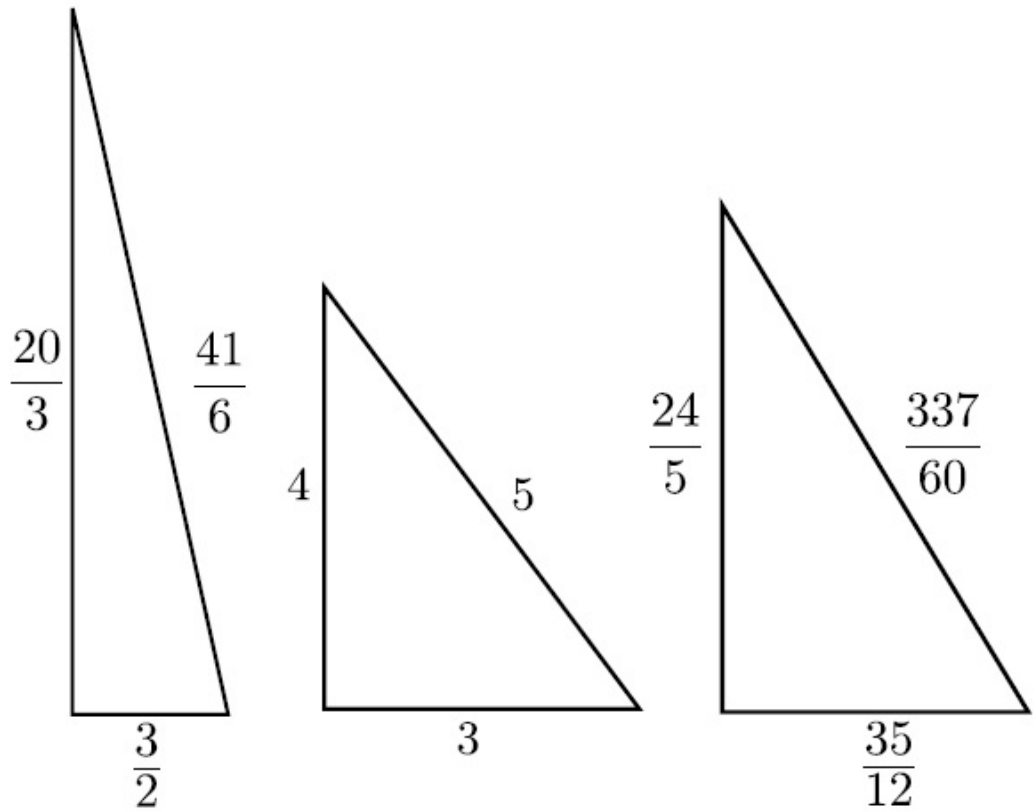


FIGURE 1. Rational right triangles with respective areas 5, 6, and 7.

A Bad Algorithm

k	ℓ	(a,b,c)	$(1/2)ab$	Squarefree part
2	1	$(3,4,5)$	6	6
4	1	$(15,8,17)$	60	15
3	2	$(5,12,13)$	30	30
6	1	$(35,12,37)$	210	210
5	2	$(21,20,29)$	210	210
4	3	$(7,24,25)$	84	21
8	1	$(63,16,65)$	504	126
7	2	$(42,28,53)$	630	70
5	4	$(9,40,41)$	180	5

Signs on $(3/2, 20/3, 41/6)$	(x, y)
$(+,+,+)$	$(25/4, 75/8)$
$(+,+,-)$	$(-4, -6)$
$(-,-,+)$	$(-4, 6)$
$(-,-,-)$	$(25/4, -75/8)$

TABLE 2. Solutions to $y^2 = x^3 - 25x$.

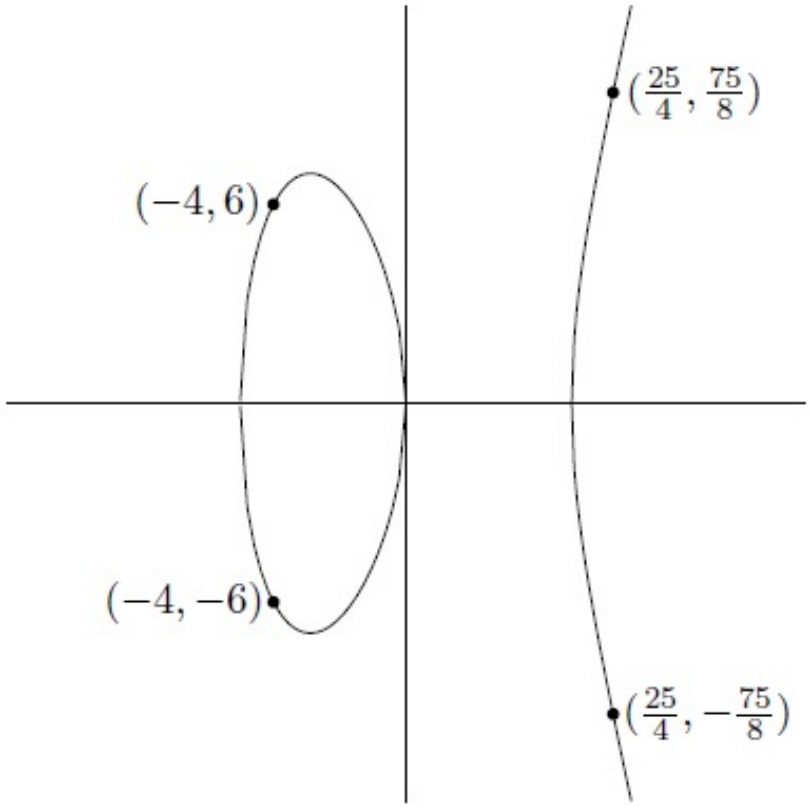


FIGURE 3. Some rational points on $y^2 = x^3 - 25x$.

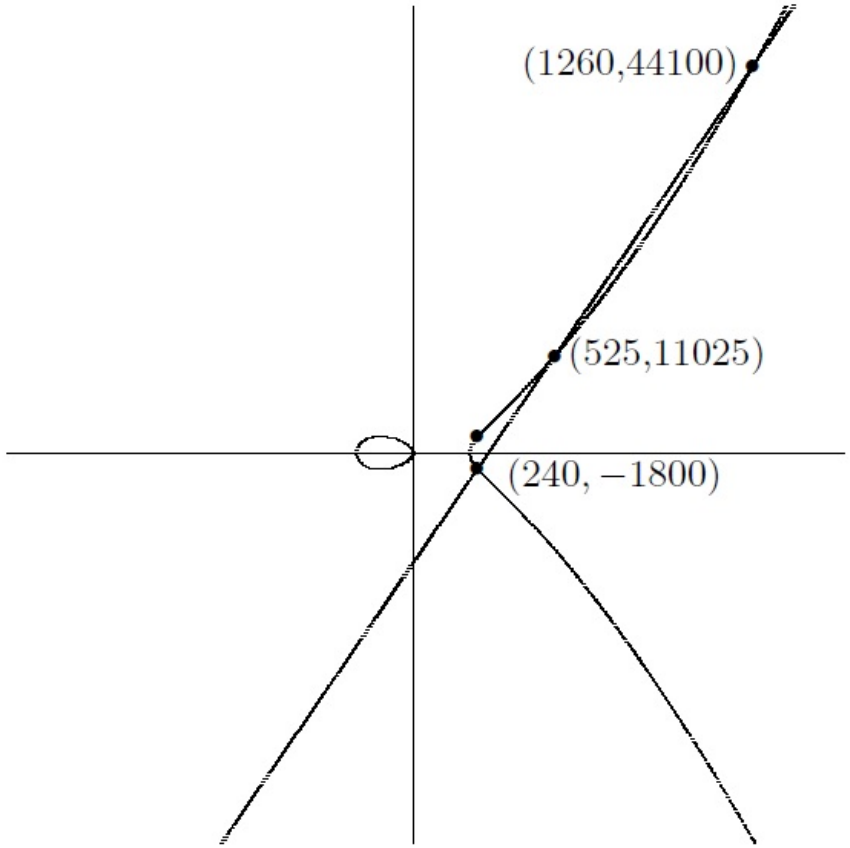


FIGURE 4. New rational point on $y^2 = x^3 - 210^2 x$ from a secant line. (Not drawn to scale.)

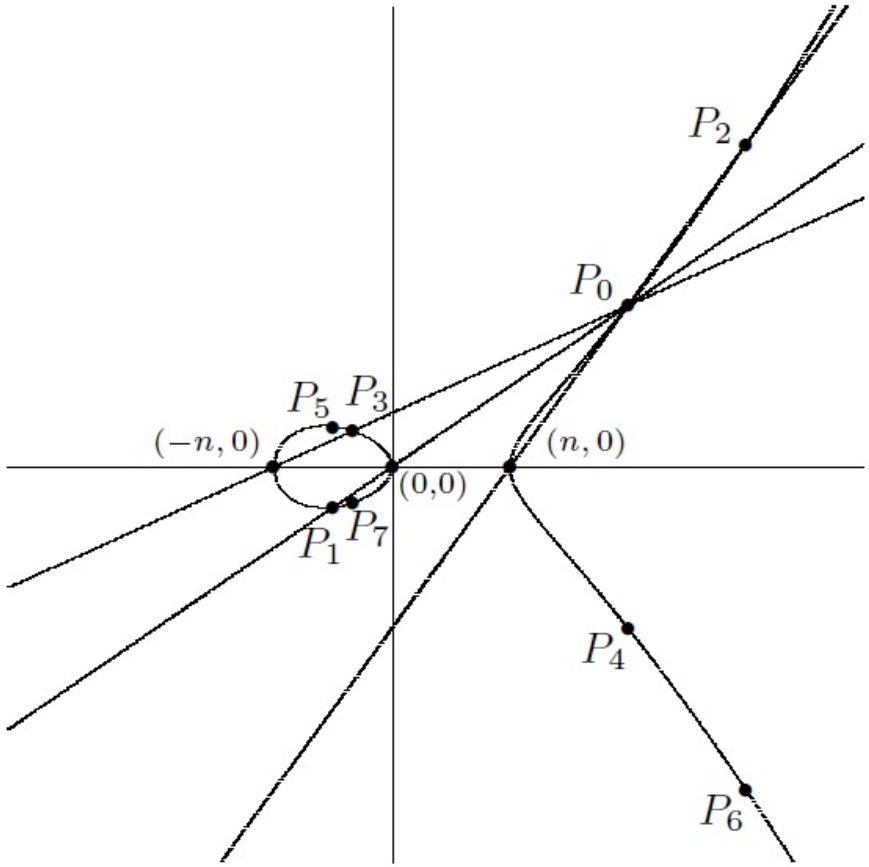


FIGURE 5. Intersecting $y^2 = x^3 - n^2x$ with lines through P_0 and $(0,0)$, $(n,0)$, $(-n,0)$, and reflected points.

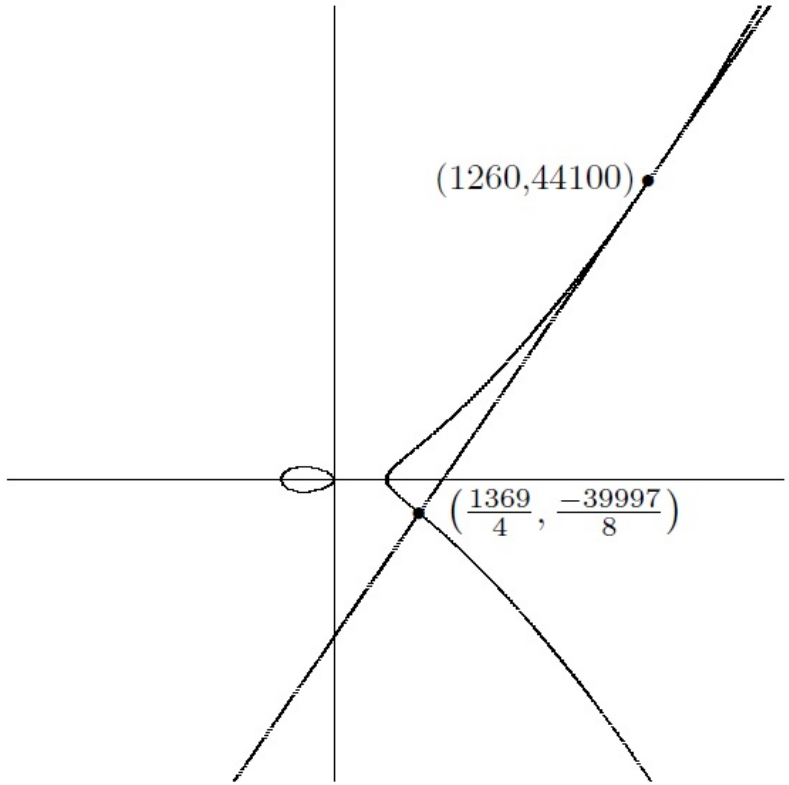


FIGURE 6. New rational point on $y^2 = x^3 - 210^2 x$ from a tangent line. (Not drawn to scale.)

First Point	Second Point	Third Point
(x, y)	$(0, 0)$	$(-n^2/x, -n^2y/x^2)$
$(x, -y)$	$(0, 0)$	$(-n^2/x, n^2y/x^2)$
(x, y)	$(n, 0)$	$(n(x+n)/(x-n), 2n^2y/(x-n)^2)$
$(x, -y)$	$(n, 0)$	$(n(x+n)/(x-n), -2n^2y/(x-n)^2)$
(x, y)	$(-n, 0)$	$(-n(x-n)/(x+n), 2n^2y/(x+n)^2)$
$(x, -y)$	$(-n, 0)$	$(-n(x-n)/(x+n), -2n^2y/(x+n)^2)$

TABLE 3. Third Intersection Point of a Line with $y^2 = x^3 - n^2x$.

Pair	Triple
(x, y)	(a, b, c)
$(x, -y)$	$(-a, -b, -c)$
$(-n^2/x, -n^2y/x^2)$	$(a, b, -c)$
$(-n^2/x, n^2y/x^2)$	$(-a, -b, c)$
$(n(x+n)/(x-n), 2n^2y/(x-n)^2)$	(b, a, c)
$(n(x+n)/(x-n), -2n^2y/(x-n)^2)$	$(-b, -a, -c)$
$(-n(x-n)/(x+n), 2n^2y/(x+n)^2)$	$(-b, -a, c)$
$(-n(x-n)/(x+n), -2n^2y/(x+n)^2)$	$(b, a, -c)$

$$(Thank)^2 + (You)^2 = (DEU)^2$$