MATH 141 Basic Calculus I, Fall 2025-2026 Recitation #10

Q 1. *Sketch the graph of the function*

$$y = x\sqrt{8 - x^2}$$

step by step giving all details: a) domain, intercepts, b) symmetries, asymptotes, c) critical points, d) intervals of increasing/ decreasing, extrema, e) inflection points and concavity.

Q 2. ***(HMW)*** *Sketch the graph of the function*

$$a)\ y = \frac{x^3 + x - 2}{x - x^2} \qquad\qquad b)\ y = \frac{1}{x} - \frac{1}{x^3}$$

step by step giving all details: a) domain, intercepts, b) symmetries, asymptotes, c) critical points, d) intervals of increasing/ decreasing, extrema, e) inflection points and concavity.

Q 3. *Evaluate the following limits.*

a) ***(HMW)*** $\displaystyle\lim_{x\to 0} \frac{2\sin x - \tan(2x)}{2e^x - 2 - 2x - x^3}$

b) $\displaystyle\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

c) $\displaystyle\lim_{x\to \infty} x^2\left(1 - \cos\frac{1}{x}\right)$

Q 4. *Use L'Hôpital's rule to find the following limits*

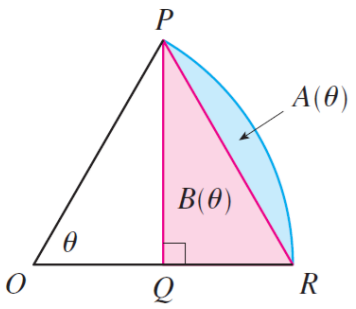
a) ***(HMW)*** $\displaystyle\lim_{t\to 0} \frac{(e^t - 1)^2}{t\sin t}$

b) $\displaystyle\lim_{x\to e^+} (\ln x)^{1/(x-e)}$

c) $\displaystyle\lim_{x\to 0^+} (\tan 2x)^x$

d) $\displaystyle\lim_{x\to 0} \Big(\csc x - \cot x\Big)$

Q 5. *The figure shows a sector of a circle with central angle θ. Let $A(\theta)$ be the area of the segment between the chord PR and the arc PR. Let $B(\theta)$ be the area of the triangle PQR. Find* $\lim_{\theta\to 0^+} A(\theta)/B(\theta)$.



Q 6. *For what values of a and b is the following equation true?*

a) $\lim_{x\to 0}\left(\dfrac{\sin 2x}{x^3} - a + \dfrac{b}{x^2}\right) = 0$

b) ***(HMW)*** $\lim_{x\to\infty}\left(\dfrac{x+a}{x-a}\right)^x = e$

Q 7. *Find the point on the parabola $y^2 = 2x$ that is the closest to the point $P = (1, 4)$.*

Q 8. ***(HMW)*** *A rectangle is to be inscribed in the ellipse*

$$4x^2 + y^2 = 4.$$

What should the dimensions of the rectangle be to maximize its area? What is the maximum area?

Q 9. *Find the most general antiderivative for each function*

a) $-x^{-3} + 2x - 3\sqrt{5x}$

b) $\cos\dfrac{\pi x}{2} + \sin 3x + e^{4x/3}$

c) $\sec(5x)\Big(\sec(5x) + \tan(5x)\Big)$

Indefinite Integrals

A special symbol is used to denote the collection of all antiderivatives of a function f.

DEFINITION The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x; it is denoted by

$$\int f(x)\,dx.$$

The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

Q 10. *Evaluate the indefinite integral or find the most general antiderivative*

a) $\displaystyle\int \left(t^{-1/3} + e^{-2t} + \frac{1}{t^{5/4}} \right) dt$

b) $\displaystyle\int \left(7 \sin \frac{\theta}{3} + 3 \cos 5\theta \right) d\theta$

c) $\displaystyle\int \left(\frac{1}{x} - \frac{5}{1+x^2} \right) dx$