
Q 1. *Estimate the area under the graph of the function* $f(x) = 1/x$ *on* $[1, 5]$ *using*

a) *a lower sum with four rectangles of equal width.*

b) *an upper sum with four rectangles of equal width.*

c) *the midpoint rule with four rectangles of equal width.*

Q 2. *For the functions below, find a formula for the Riemann sum obtained by dividing the interval* $[a, b]$ *into* n *equal subintervals and using the right-hand endpoint for each* c_k*. Then take a limit of each of these sums as* $n \to \infty$ *to calculate the area under the curve over* $[a, b]$*.*

a) $f(x) = x^2 - x^3$ *over the interval* $[-1, 0]$.

b) $g(x) = 3x + 1$ *over the interval* $[0, 3]$.

Q 3. *Suppose that* f *and* h *are integrable and that*

$$\int_1^9 f(x)\,dx = -1, \qquad \int_7^9 f(x)\,dx = 5, \qquad \int_7^9 h(x)\,dx = 4.$$

Find

a) $\displaystyle\int_7^9 [2f(x) - 3h(x)]\,dx$

b) $\displaystyle\int_1^7 f(x)\,dx$

Q 4. *Use the known area formulas to evaluate the following integrals.*

a) $\displaystyle\int_{-1}^1 (1 + \sqrt{1 - x^2})\,dx$

b) $\displaystyle\int_{-3}^3 (4 - |x|)\,dx$

Q 5. *Find the average value of the function* $f(x) = t^2 - t$ *over the interval* $[-2, 1]$.

Q 6. *Find the total area between the region and the* x*-axis for the following functions*

a) $f(x) = x^3 - 3x^2 + 2x, \;\; 0 \le x \le 2.$

b) $g(x) = \sec x \tan x, \;\; 0 \le x \le \pi/4.$

Q 7. *What values of* a *and* b*, with* $a < b$*, minimize the value of*

$$\int_a^b (x^4 - 2x^2)\,dx?$$

Q 8. *Find the following derivatives*

a) $\dfrac{d}{dt}\displaystyle\int_{\sqrt{t}}^{0}\left(x^4+\dfrac{3}{\sqrt{1-x^2}}\right)dx$

b) $\dfrac{d}{dx}\displaystyle\int_{\sin x}^{\cos x}\ln(t^2+1)\,dt.$

Q 9. *Evaluate the following integrals:*

a) $\displaystyle\int_{\pi/2}^{0}\dfrac{1+\cos 2t}{2}\,dt$

b) $\displaystyle\int_{1}^{8}\dfrac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}}\,dx$

Q 10. *Evaluate the following limit:*

$$\lim_{x\to 0^+}\frac{\displaystyle\int_0^{x^2}\sin\sqrt{t}\,dt}{x^2}$$