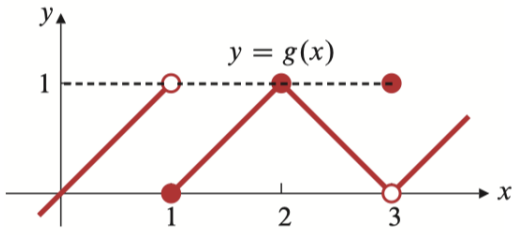

Q 1. *Prove the limit statements*

a) $\lim_{x\to 1} f(x) = 1 \quad \textit{if} \quad f(x) = \begin{cases} x^2, & x \neq 1 \\ 1, & x = 1 \end{cases}$

b) $\lim_{x\to 0^+} \dfrac{x}{x+2} = 0$

Solution:

Q 2. *For the function $y = g(x)$ graphed below, find each of the following limits or explain why it does not exist.*



a) $\lim_{x \to 1} g(x)$

b) $\lim_{x \to 2} g(x)$

c) $\lim_{x \to 3} g(x)$

Solution:

Q 3. *Evaluate the following limits*

a) $\lim_{x \to -2^-} (x+3)\dfrac{|x+2|}{x+2}$

b) $\lim_{x \to \frac{\pi}{2}^+} \dfrac{|cos(x)|}{x - \frac{\pi}{2}}$

Solution:

Q 4. *Find the limits*

a) $\lim_{t \to 0} \dfrac{2t}{\tan(t)}$

b) $\lim_{t \to 0} \dfrac{t + t\cos(t)}{\sin(t)\cos(t)}$

Solution:

Q 5. *Find the limits*

a) $\lim_{x\to\infty}\left(\dfrac{2-3x^3+5x^2}{1-x^2-4x^3}\right)$

b) $\lim_{x\to\infty}\dfrac{2\sqrt{x}+x^{-1}}{3x-7}$

c) $\lim_{x\to-\infty}\dfrac{x-3}{\sqrt{4x^2+25}}$

Solution:

Q 6. *Find the limits*

a) $\lim_{x \to -8^+} \dfrac{2x}{x+8}$

b) $\lim_{x \to 0} \dfrac{-1}{x^2(x+1)}$

c) $\lim_{\theta \to 0^-} (1 + \csc(\theta))$

Solution:

Definition 1. ***(Horizontal Asymptote)*** *$y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either*

$$\lim_{x\to\infty} f(x) = b \quad or \quad \lim_{x\to-\infty} f(x) = b.$$

Definition 2. ***(Oblique Asymptote)*** *If the degree of the numerator of a rational function is 1 greater than the degree of the denominator, the graph has an oblique or slant line asymptote. We find an equation for the asymptote by dividing numerator by denominator to express f as a linear function plus a remainder that goes to zero as $x \to \pm\infty$.*

Definition 3. ***(Vertical Asymptote)*** *A line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either*

$$\lim_{x\to a^+} f(x) = \pm\infty \quad or \quad \lim_{x\to a^-} f(x) = \pm\infty$$

Q 7. *Find the asymptotes of the given function*

$$f(x) = \frac{5 + 4x}{x + 3}$$

and examine behaviour.
Solution: