
Definition 1. ***(Horizontal Asymptote)*** *$y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either*

$$\lim_{x\to\infty} f(x) = b \quad or \quad \lim_{x\to-\infty} f(x) = b.$$

Definition 2. ***(Oblique Asymptote)*** *If the degree of the numerator of a rational function is 1 greater than the degree of the denominator, the graph has an oblique or slant line asymptote. We find an equation for the asymptote by dividing numerator by denominator to express f as a linear function plus a remainder that goes to zero as $x \to \pm\infty$.*

Definition 3. ***(Vertical Asymptote)*** *A line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either*

$$\lim_{x\to a^+} f(x) = \pm\infty \quad or \quad \lim_{x\to a^-} f(x) = \pm\infty$$

Q 1. *Find the asymptotes of the given functions and examine behaviors.*

a) $f(x) = \dfrac{2x^2 + x - 1}{x^2 + x - 2}$

b) $f(x) = \dfrac{x^2 - 1}{2x + 4}$

Q 2. *Find the asymptotes of*

$$y = \frac{8 - e^x}{2 + e^x}.$$

Q 3. *Show that*

$$\lim_{x\to-\infty} \frac{x}{x+1} = 1.$$

Q 4. *Evaluate the following limits.*

a) $\displaystyle\lim_{x\to-\infty} \left(\frac{1 - x^3}{x^2 + 7x}\right)^5$

b) $\displaystyle\lim_{x\to-\infty} \left(\frac{x^2 + x - 1}{8x^2 - 3}\right)^{1/3}$

c) $\displaystyle\lim_{x\to\frac{\pi}{2}^+} \frac{|1 - \sin(x)|}{\sin(x) - 1}$

Q 5. *Find*

$$\lim \left(\frac{1}{x^{2/3}} + \frac{2}{(x-1)^{1/3}}\right)$$

as

a)$x \to 1^+$ **b)**$x \to 1^-$

Q 6. *Find* $\lim_{x\to\infty}(\sqrt{x^2+25}-\sqrt{x^2-1})$.

Dominant Terms

By using long division, we can rewrite the function

$$f(x)=\frac{x^2-3}{2x-4}$$

as a linear function plus a reminder term:

$$f(x)=\left(\frac{x}{2}+1\right)+\left(\frac{1}{2x-4}\right).$$

This tells us immediately that

For x large:	$f(x)\approx\frac{x}{2}+1$	$\frac{1}{2x-4}$ is near 0.
For x near 2:	$f(x)\approx\frac{1}{2x-4}$	This term is very large in absolute value.

If we want to know how f behaves, this is one possible way to find out. It behaves like $y=(x/2)+1$ when $|x|$ is large and the contribution of $1/(2x-4)$ to the total value of f is insignificant. It behaves like $1/(2x-4)$ when x is so close to 2 that $1/(2x-4)$ makes the dominant contribution.

We say that $(x/2)+1$ **dominates** when x approaches ∞ or $-\infty$, and we say that $1/(2x-4)$ dominates when x approaches 2. **Dominant terms** like these help us predict a function's behavior.

Q 7. *Let* $f(x) = 3x^4 - 2x^3 + 3x^2 - 5x + 6$ *and* $g(x) = 3x^4$*. Show that although* f *and* g *are quite different for numerically small values of* x*, they behave similarly for very large* $|x|$*, in the sense that their ratios approach* 1 *as* $x \to \infty$ *or* $x \to -\infty$*.*

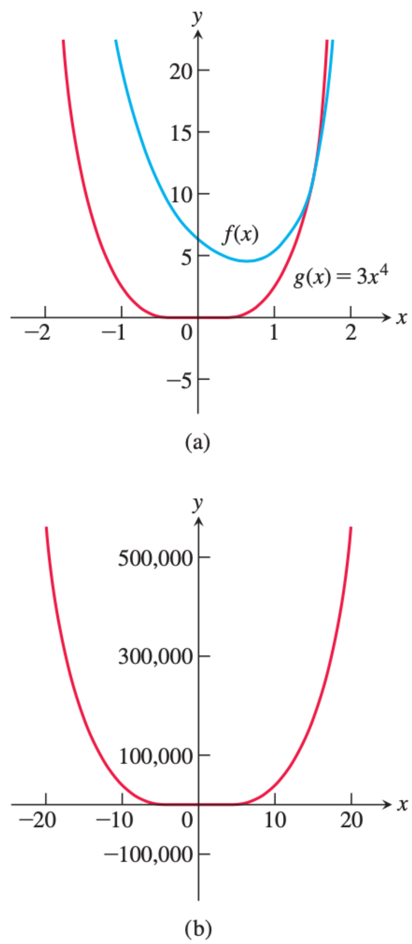


FIGURE 2.55 The graphs of f and g are (a) distinct for $|x|$ small, and (b) nearly identical for $|x|$ large (Example 20).

DEFINITIONS Let c be a real number that is either an interior point or an endpoint of an interval in the domain of f.

The function f is **continuous at c** if

$$\lim_{x \to c} f(x) = f(c).$$

The function f is **right-continuous at c** (or **continuous from the right**) if

$$\lim_{x \to c^+} f(x) = f(c).$$

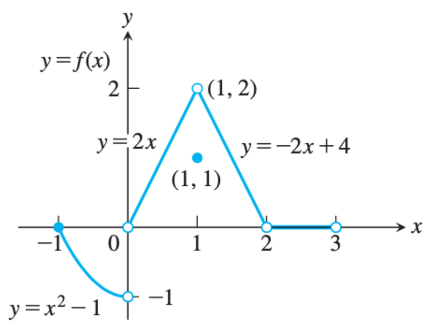
The function f is **left-continuous at c** (or **continuous from the left**) if

$$\lim_{x \to c^-} f(x) = f(c).$$

Q 8. *Let*

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$

graphed in the accompanying figure.



a) Is f is continuous at $x = -1$?

b) Is f is continuous at $x = 1$?

c) Is f is continuous at $x = 2$?