
Q 1. *If the given function $f(x)$ is continuous everywhere, find the values of a and b*

$$f(x) = \begin{cases} \sqrt{3-x} & \text{if } x < 2, \\ \dfrac{ax-b}{3} & \text{if } 2 \le x \le 5, \\ x+a-b & \text{if } x > 5. \end{cases}$$

Q 2. *Determine the points where the following function is discontinuous:*

$$f(t) = \begin{cases} \dfrac{t+1}{t^2-4} & \text{if } t < -1, \\ t^3+t & \text{if } -1 \le t \le 1, \\ \dfrac{t-1}{t^2-4} & \text{if } t > 1. \end{cases}$$

Q 3. *Prove that the equation $x2^x = 1$ has at least one positive root less than unity.*

Q 4. *Show that the equation $x^8 + x - 1 = 0$ has two real roots.*

Q 5. *Continuous Extension to a Point*

Sometimes the formula that describes a function f does not make sense at a point $x = c$. It might nevertheless be possible to extend the domain of f to include $x = c$, creating a new function that is continuous at $x = c$.

More generally, a function (such as a rational function) may have a limit at a point where it is not defined. If $f(c)$ is not defined, but exists, we can define a new function $F(x)$ by the rule

$$F(x) = \begin{cases} f(x) & \text{if } x \text{ is in the domain of } f \\ L & \text{if } x = c. \end{cases}$$

*The function F is continuous at $x = c$. It is called the **continuous extension of f to** $x = c$. For rational functions f, continuous extensions are often found by canceling common factors in the numerator and denominator.*

Show that

$$f(x) = \frac{x^2+x-6}{x^2-4}, \quad x \neq 2.$$

has a continuous extension to $x = 2$, and find that extension.

Q 6. *Use the formal definition of limit to prove that the function has a continuous extension to the given value of x.*

$$f(x) = \frac{x^2 - 1}{x + 1}, \quad x = -1.$$

Q 7. *Find the equation of tangent line to the curve at the given point*

$$f(x) = \frac{8}{\sqrt{x - 2}}, \qquad x = 6.$$

Q 8. *Does the graph of*

$$f(x) = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$$

have a vertical tangent line at the origin? Give reasons for your answer.

Q 9.

Definition 1. ***(Derivative)*** *The derivative of the function $f(x)$ with respect to the variable x is the function f' whose value at x is*

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h},$$

provided the limit exists.

Using the definition, calculate the derivative of the following function and find the values of the derivative as specified.

$$f(t) = \frac{1 - t}{2t}, \qquad f'(-1), \; f'(\sqrt{2}).$$