

Q 1. Find the derivatives of the following functions:

a) $y = (x^2 + 3e^x + 1)(\sec x + 5 + \frac{1}{x})$

b) $x = \frac{t^2 - \tan t}{t + \sin(t) - 2}$

Q 2. Free fall from the Tower of Pisa Had Galileo dropped a cannonball from the Tower of Pisa, 56m above the ground, the ball's height above the ground t seconds into the fall would have been $s = 56 - 4.9t^2$

a) What would have been the ball's velocity, speed, and acceleration at time t ?

b) About how long would it have taken the ball to hit the ground?

c) What would have been the ball's velocity at the moment of impact?

Q 3. By computing the first few derivatives and looking for a pattern, find the following derivative:

$$\frac{d^{73}}{dx^{73}}(x \sin x).$$

Q 4. For the given $y = f(u)$ and $u = g(x)$, find $dy/dx = f'(g(x))g'(x)$.

a) $y = -\sec u, u = \frac{1}{x} + 7x$

b) $y = e^u, u = 4\sqrt{x} + x^2$

Q 5. Find the first and second derivatives of the following function:

$$y = \left(1 + \tan^2 \left(\frac{t}{12} \right) \right)^3$$

Q 6. Consider the function

$$f(x) = \begin{cases} x \sin \left(\frac{1}{x} \right), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

a) Show that f is continuous at $x = 0$.

b) Determine f' for $x \neq 0$.

c) Show that f is not differentiable at $x = 0$.

Q 7. Use implicit differentiation to find dy/dx and then d^2y/dx^2 . Write the solutions in terms of x and y only.

a) $\sin y = x \cos y - 2$

b) $e^{x^2y} = 2x + 2y$

Q 8. Find equations for the tangent line and the normal line to the cissoid of Diocles

$$y^2(2 - x) = x^3$$

at $(1, 1)$.

Q 9. Let $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1$.

Q 10. Suppose that the differentiable function $y = f(x)$ has an inverse and that the graph of f passes through the point $(2, 4)$ and has a slope of $1/3$. Find the values of df^{-1}/dx at $x = 4$.