MATH 145 Calculus for Engineering and Science I
Midterm 1

November 3th, 2025

1. For which numbers a, b, c, and d will the function

$$f(x) = \frac{ax + b}{cx + d}$$

satisfy $f(f(x)) = x$ for all x?

2. i. Show that the straight line through (a, b) with slope m is the graph of the function $f(x) = m(x - a) + b$; it is immediately clear from the point-slope form that the slope is m, and that the value of f at a is b.

   ii. For $a \neq c$, show that the straight line through (a, b) and (c, d) is the graph of the function

$$f(x) = \frac{d - b}{c - a}(x - a) + b\,.$$

3. i. Let R_θ be rotation by angle of θ. Show that $R_\theta(v) \cdot R_\theta(w) = v \cdot w$ ($\cdot$ is the dot (or scalar) product for vectors).

   ii. Let $e = (1, 0)$ be the vector of length 1 pointing along the first axis, and let $w = (\cos\theta, \sin\theta)$; this is a vector of length 1 that makes an angle of with the first axis. Calculate that

$$e \cdot w = \cos\theta\,.$$

4. Prove that $\lim_{x \to a} f(x)$ exists if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$.

5. Suppose that f is a function satisfying $|f(x)| \leq |x|$ for all x. Show that f is continuous at 0. (Notice that $f(0)$ must equal 0.)