

# MATH 145 Calculus for Engineering and Science I

## Midterm 2

December 8th, 2025

1. Find the derivative of the following function

$$F(x) = \int_x^b \frac{1}{1 + t^2 + \sin^2 t} dt$$

2. Suppose that  $f$  is differentiable with derivative  $f'(x) = (1 + x^3)^{-1/2}$ . Show that that  $g = f^{-1}$  satisfies  $g''(x) = \frac{3}{2}g(x)^2$ .

3. Show that  $f(x) = \frac{ax+b}{cx+d}$  is one-to-one if  $ad - bc \neq 0$ , and find  $f^{-1}$ .

4. Prove that

$$\int_{ca}^{cb} f(t)dt = c \int_a^b f(ct)dt$$

(Hint: If  $P = \{t_0, \dots, t_n\}$  and  $P' = \{ct_0, \dots, ct_n\}$  of  $[a, b]$ , then  $m_i = \inf\{f(ct) : t_{i-1} \leq t \leq t_i\} = \inf\{f(t) : ct_{i-1} \leq t \leq ct_i\}$ .)

5. Find  $(f^{-1})'(0)$  if

i.  $f(x) = \int_0^x (1 + \sin(\sin t)) dt$

ii.  $f(x) = \int_1^x \cos(\cos t) dt$