MATH 145: Calculus for Engineering and Science I
Recitation 4 Solution Key

November 6th, 2025

1. For each of the following polynomial functions f, find an integer n such that $f(x) = 0$ for some x between n and $n + 1$.

1.1 $f(x) = x^3 - x + 3$

Solution: We evaluate the function at integer values to identify a sign change.

$$f(-1) = (-1)^3 - (-1) + 3 = 3 > 0$$
$$f(-2) = (-2)^3 - (-2) + 3 = -8 + 2 + 3 = -3 < 0$$

Since $f(x)$ is a polynomial, it is continuous on $\mathbb{R}$. Since $f(-2) < 0$ and $f(-1) > 0$, by the Intermediate Value Theorem, there exists a root c such that $f(c) = 0$ with $c \in (-2, -1)$.

Answer: $n = -2$

1.2 $f(x) = 4x^2 - 4x + 1$

Solution: We can factor this quadratic expression directly

$$4x^2 - 4x + 1 = (2x - 1)^2$$

Setting $f(x) = 0$

$$(2x - 1)^2 = 0 \implies 2x = 1 \implies x = 0.5$$

The root $x = 0.5$ lies strictly between the integers 0 and 1.

Answer: $n = 0$

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2. Find the least upper bound (lub) and the greatest lower bound (glb) of the following sets.

2.1 $\{1/n : n \in \mathbb{N}\}$

Solution: Let $S = \{1, \frac{1}{2}, \frac{1}{3}, \dots\}$.

- **lub:** The sequence is strictly decreasing. The maximum element is occurring at $n = 1$, which is 1. Thus, $\text{lub}(S) = 1$.
- **glb:** Since $n > 0$, $\frac{1}{n} > 0$ for all n, so 0 is a lower bound. As $n \to \infty$, $\frac{1}{n}$ approaches 0. Thus, there is no lower bound greater than 0. $\text{glb}(S) = 0$.

2.2 $\{x : x^2 + x + 1 \geq 0\}$

Solution: We analyze the quadratic inequality. The discriminant of $x^2 + x + 1$ is

$$\Delta = b^2 - 4ac = 1^2 - 4(1)(1) = -3$$

Because $\Delta < 0$ and the leading coefficient is positive ($a = 1$), the parabola opens upwards and lies entirely above the x-axis. Therefore, $x^2 + x + 1 > 0$ for all $x \in \mathbb{R}$.

The set is equivalent to $\mathbb{R}$ (all real numbers), which is unbounded.

Answer: lub and glb do not exist.