MATH 145 Calculus for Engineering and Science I

Recitation 7

- KEY -

December 2th, 2025

1. Evaluate the following integral without any computations
   1. $\int_{-1}^{1} x^3\sqrt{1-x^2}dx$
2. Prove that
$$\int_0^x \frac{\sin t}{t+1}dt > 0\,, \quad \forall x > 0$$
3. Find the areas of the regions bounded by
   1. the graphs of $f(x) = x^2$ and $g(x) = -x^2$ and the vertical lines through $(-1, 0)$ and $(1, 0)$.
   2. the graphs of $f(x) = x^2$ and $g(x) = 1 - x^2$
4. Find a function g such that
$$\int_0^x tg(t)dt = x + x^2$$
5. Find $F'(x)$ if $F(x) = \int_0^x xf(x)dt$ (The answer is not $xf(x)$!).

1) Let $h(x) = x^3\sqrt{1-x^2}$.

$\Rightarrow h(-x) = (-x)^3\sqrt{1-(-x)^2} = -x^3\sqrt{1-x^2} = -h(x)$

$\Rightarrow$ h is odd

$$\Rightarrow \int_{-1}^{1} h(x)dx = 0$$

2) Let $A_n := \int_{(n-1)\pi}^{n\pi} \frac{\sin t}{t+1} dt$

Observe that

$$A_1 = \int_0^{\pi} \frac{\sin t}{t+1} dt > 0 \quad \text{as} \quad \frac{\sin t}{t+1} > 0 \quad \forall t \in (0, \pi)$$

$$A_2 = \int_{\pi}^{2\pi} \frac{\sin t}{t+1} dt < 0 \quad \text{as} \quad \frac{\sin t}{t+1} < 0 \quad \forall t \in (\pi, 2\pi)$$

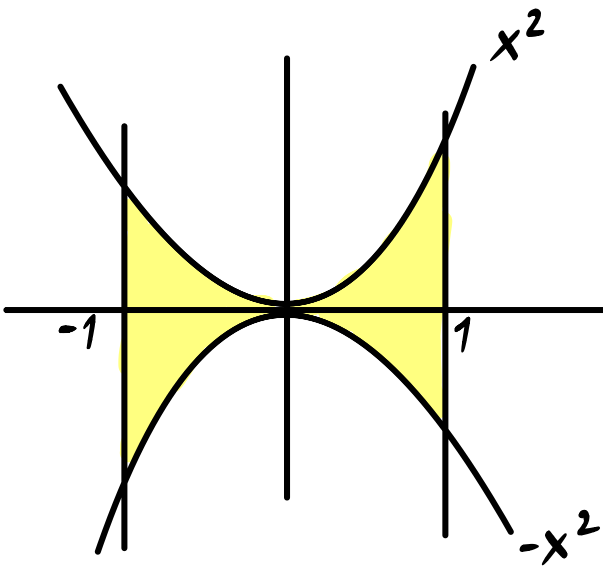
In general, $A_n > 0$ if n is odd
$A_n < 0$ if n is even

As $t+1 \longrightarrow \infty$ when $t \to \infty$, $|A_1| > |A_2| > \dots$

$$\int_0^x \frac{\sin t}{t+1}\,dt = \underbrace{\underbrace{(A_1 + A_2)}_{>0} + \cdots + \underbrace{(A_{k-1} + A_k)}_{>0} + \int_{k\pi}^{x} \frac{\sin t}{t+1}\,dt}_{>0}$$

even if $\int_{k\pi}^{x} \frac{\sin t}{t+1}\,dt < 0$.

3)

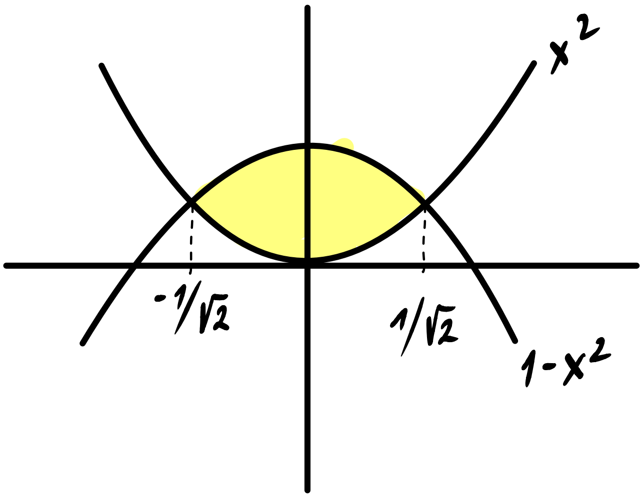


$$A = \int_{-1}^{1} x^2 - (-x^2)\,dx$$

$$= \int_{-1}^{1} 2x^2\,dx$$

$$= \frac{2}{3}x^3 \Big|_{-1}^{1}$$

$$= \frac{4}{3}$$



$$x^2 = 1 - x^2$$
$$\Rightarrow 2x^2 = 1$$
$$\Rightarrow x^2 = 1/2$$
$$\Rightarrow x = \pm 1/\sqrt{2}$$

$$A = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left[(1-x^2) - x^2\right] dx = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1-2x^2)\, dx$$

$$= x - \frac{2}{3}x^3 \Big|_{-1/\sqrt{2}}^{1/\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{3}$$

4) $\frac{d}{dx}\left(\int_0^x tg(t)\,dt\right) = \frac{d}{dx}\left(x+x^2\right)$

$\Rightarrow \quad xg(x) = 1 + 2x$

by FTC
(need to
check if
$tg(t)$ is continuous on $[0,x)$

$\Rightarrow \quad g(x) = \frac{1}{x} + 2$

5) $F(x) = \int_0^x xf(x)\,dt$

$= xf(x)\int_0^x dt$

$= x^2 f(x)$

$F'(x) = 2xf(x) + x^2 f'(x)$