MATH 145 Calculus for Engineering and Science I
Recitation 10

January 5th, 2026

1. Let $a_1 = 1$ and $a_{n+1} = \sqrt{1 + 2a_n}$ $(n = 1, 2, 3, \cdots)$. Show that $\{a_n\}$ is increasing and bounded above. (Hint: Show that 3 is an upper bound.) Hence, conclude that the sequence converges, and find its limit.

2. Let $a_n = (1+1/n)^n$ so that $\ln a_n = n \ln(1+1/n)$. Use the properties of the logarithm function to show that (a) $\{a_n\}$ is increasing and (b) e is an upper bound for $\{a_n\}$.

3. Obtain a simple expression for the partial sum s_n of the series $\lim_{n=1}^{\infty}(-1)^n$, and use it to show that the series diverges.

4. Use the integral test to show that $\sum_{n=1}^{\infty} 1/(1 + n^2)$ converges. Show that the sum s of the series is less than $\pi/2$.

5. Use the root test to show that $\sum_{n=1}^{\infty} 2^{n+1}/n^n$ converges.