MATH 145 Calculus for Engineering and Science I
Recitation 2

October 20th, 2025

1. Find the domain of the functions defined by the following formulas.
   1. $\sqrt{1-\sqrt{1-x^2}}$
   2. $\dfrac{x+1}{x-1}+\dfrac{x-1}{x-2}$
   3. $\sqrt{1-x+\sqrt{x-2}}$
2. A function f is called **even** if $f(x)=f(-x)$ and **odd** if $f(x)=-f(-x)$.

   For example:
   $$f(x)=x^2, \quad f(x)=|x|, \quad f(x)=\cos x$$
   are even functions, while
   $$f(x)=x, \quad f(x)=\sin x$$
   are odd functions.

   (a) Determine whether $f+g$ is even, odd, or not necessarily either, in the four cases obtained by choosing f even or odd, and g even or odd.

   (b) Do the same for the product $f \cdot g$.

   (c) Do the same for the composition $f \circ g$.
3. Draw the set of all points (x, y) satisfying each of the following conditions.
   1. $|x|+|y|=1$
   2. $x^2+y^2=0$
   3. $x^2-2x+y^2=4$
4. Given vectors $\mathbf{v}=(v_1, v_2)$ and $\mathbf{w}=(w_1, w_2)$, we define the number
   $$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2,$$
   which is called the **dot product** or **scalar product** of $\mathbf{v}$ and $\mathbf{w}$.

(a) Given a vector $\mathbf{v}$, find a vector $\mathbf{w}$ such that $\mathbf{v} \cdot \mathbf{w} = 0$. Describe the set of all such vectors $\mathbf{w}$.

(b) Show that:

$$\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v},$$

$$\mathbf{v} \cdot (\mathbf{w} + \mathbf{z}) = \mathbf{v} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{z},$$

and that

$$a(\mathbf{v} \cdot \mathbf{w}) = (a\mathbf{v}) \cdot \mathbf{w} = \mathbf{v} \cdot (a\mathbf{w}).$$

5. Consider a cylinder with a generator perpendicular to the horizontal plane; the only requirement for a point (x, y, z) to lie on this cylinder is that (x, y) lies on a circle:

$$x^2 + y^2 = C^2$$

Show that the intersection of a plane with this cylinder can be described by an equation of the form

$$(\alpha x + \beta)^2 + y^2 = C^2 \,.$$

What are the possibilities?